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SCATTERING OF ELECTROMAGNETIC WAVES BY AN IONIZED WAKE IN THE FORM OF A PARABOLOID OF REVOLUTION

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SUMMARY

Trajectories of rays are found in an ionized paraboloid of revolution, in which the concentration of electrons is dependent upon two coordinates. The flux density decrease at the expense of energy absorption in the ionized medium is determined. The conditions for radar detection of the wake are ascertained. The magnitude of the wake's effective reflecting surface is computed for a rather arbitrary distribution function of free element concentration.

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INTRODUCTION

As a result of an ionized source's motion in space, there forms behind it an ionized wake. The distribution of concentration of free electrons in the wake varies, as a rule, along the wake's axis and in directions perpendicular to it. The most appropriate model of an ionized wake may apparently be represented with the help of a parabolic system of coordinates.

The index of refraction of wake's ionized medium can appropriately be written in the form

$$n^2 = 1 - \frac{f(\xi_1)}{h^2(\xi_1, \xi_2)}. \qquad (1)$$

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* RASSEYANIYE ELEKTROMAGNITNYKH VOLN IONIZIROVANNYM SLEDOM V VIDE PARABOLOIDA VRASHCHENIYA.

Here $f(\xi_1)$ is an arbitrary function of ξ_1; $h^2(\xi_1, \xi_2)$ is a function of ξ_1 and ξ_2 which must be so selected that the eikonal equation may be resolved by the method of variable separation.

In the following we shall consider that the source emitting the electromagnetic waves is outside the limits of the wake.

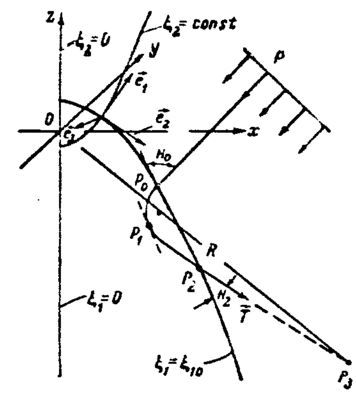

Fig. 1

1. — SOLUTION OF THE EIKONAL EQUATION

The eikonal equation

$$(\nabla U)^2 = n^2, \tag{2}$$

of which the deduction may be found in reference [1], may be written in the parabolic system of coordinates as follows:

$$\frac{U_{\xi_1}{}^2}{h_1{}^2} + \frac{U_{\xi_2}{}^2}{h_2{}^2} + \frac{U_{\xi_3}{}^2}{h_3{}^2} = 1 - \frac{f(\xi_1)}{h^2(\xi_1, \xi_2)}, \tag{3}$$

where h_1, h_2, h_3 are Lamé coefficients :

$$h_1 = h_2 = \sqrt{\xi_1{}^2 + \xi_2{}^2},$$
$$h_3 = \xi_1\xi_2 / \sqrt{1 - \xi_3{}^2}.$$

If we assume $h = h_1 = h_2$, the general solution of the equation (3)

$$U = \int_{\xi_{10}}^{\xi_1} \sqrt{-a_1{}^2 - \frac{a_3{}^2}{\xi_1{}^2} + \xi_1{}^2 - f}\, d\xi_1 +$$
$$+ \int_{\xi_{20}}^{\xi_2} \sqrt{\xi_2{}^2 - \frac{a_3{}^2}{\xi_2{}^2} + a_1{}^2}\, d\xi_2 + \int_{\xi_{30}}^{\xi_3} \sqrt{\frac{a_3{}^2}{1 - \xi_3{}^2}}\, d\xi_3 + a_2 =$$
$$= \psi(\xi_1, \xi_2, \xi_3, a_1, a_3) + a_2, \tag{4}$$

where a_1, a_2, a_3 are the integration constants.

The equation (4) represents the surfaces of equal phases. It is well known [2] that when (4) is determined, the equations for the trajectory of the rays are determined by the Jacobi theorem

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$$\frac{\partial\psi(\xi_1, \xi_2, \xi_3, a_1, a_3)}{\partial a_i} = C_i, \quad i = 1, 3. \tag{5}$$

Here the constants a_1, a_3, C_1, C_3 determine the point through which the ray passes in a preassigned direction.

Effecting the differentiation of (5), we shall obtain a system of equations which will allow the finding of the trajectory of the rays in the wake :

$$-\int_{\xi_{10}}^{\xi_1} \frac{a_1\, d\xi_1}{\sqrt{\xi_1^2 - a_1^2 - \frac{a_3^2}{\xi_1^2} - f}} + \int_{\xi_{20}}^{\xi_2} \frac{a_1\, d\xi_2}{\sqrt{\xi_2^2 - \frac{a_3^2}{\xi_2^2} + a_1^2}} = C_1,$$

$$-\int_{\xi_{10}}^{\xi_1} \frac{a_3\, d\xi_1}{\xi_1\sqrt{\xi_1^4 - \xi_1^2 a_1^2 - a_3^2 - f\xi_1^2}} - \int_{\xi_{20}}^{\xi_2} \frac{a_3\, d\xi_2}{\xi_2\sqrt{\xi_2^4 + a_1^2\xi_2^2 - a_3^2}} +$$

$$+ \int_{\xi_{30}}^{\xi_3} \frac{d\xi_3}{\sqrt{1-\xi_3^2}} = C_3.$$

We shall choose the constants C_1 and C_2 from the condition that the points $P_0(\xi_{10}, \xi_{20}, \xi_{30})$ lay on the trajectory of the ray; then

$$-\int_{\xi_{20}}^{\xi_1} \frac{\xi_1\, d\xi_1}{\sqrt{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}} + \int_{\xi_{20}}^{\xi_2} \frac{\xi_2\, d\xi_2}{\sqrt{\xi_2^4 + \xi_2^2 a_1^2 - a_3^2}} = 0,$$

$$\int_{\xi_{10}}^{\xi_1} \frac{a_3\, d\xi_1}{\xi_1\sqrt{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}} + \int_{\xi_{20}}^{\xi_2} \frac{a_3\, d\xi_2}{\xi_2\sqrt{\xi_2^4 + a_1^2\xi_2^2 - a_3^2}} = \tag{6}$$

$$= \int_{\xi_{30}}^{\xi_3} \frac{d\xi_3}{\sqrt{1-\xi_3^2}}$$

The constants a_1 and a_2 define the direction of motion of the ray. At the point P_o, lying on the boundary of the ray, of which the equation is

$$\xi_1 = \xi_{10},$$

the direction of the ray incident upon the wake must coincide with the direction of the ray propagating in the wake. The constants a_1 and a_3 are determined from this condition, similarly to the way it was done in [5]:

$$a_1^2 = (\xi_{10}^2 - \xi_{20}^2)\cos^2\alpha - \xi_{10}\xi_{20}\xi_{30}\sin 2\alpha,$$
$$a_3^2 = (1 - \xi_{30}^2)\,\xi_{10}^2\xi_{20}^2\cos\alpha. \tag{7}$$

The integrals entering in the system of equations (6), which are dependent on ξ_2 and ξ_3, may be easily computed. For certain forms of the function $f(\xi_1)$ the integrals depending on ξ_1 are also easily computed. For further calculations we shall represent (6) in the form

$$F^{(1)} = 0, \quad F^{(2)} = 0, \tag{8}$$

and we shall find the equation of the vector $\vec{T}$, tangent to ray trajectory in an arbitrary point of the wake. The vector $\vec{T}$ is determined as the vectorial product of gradients of functions $F^{(1)}$ and $F^{(2)}$:

$$\vec{T} = [\nabla F^{(1)}, \nabla F^{(2)}] = hF^{(1)}_{\xi_2}F^{(2)}_{\xi_3}\vec{e}_{\xi_1} - hF^{(1)}_{\xi_1}F^{(2)}_{\xi_3}\vec{e}_{\xi_2} + \\ + h_3(F^{(1)}_{\xi_1}F^{(2)}_{\xi_2} - F^{(2)}_{\xi_1}F^{(1)}_{\xi_2})\vec{e}_{\xi_3}, \tag{9}$$

where $\vec{e}_{\xi_1}, \vec{e}_{\xi_2}, \vec{e}_{\xi_3}$ are the orts of the coordinate system $F^{(1)}_{\xi_1} = \partial F^{(1)}/\partial\xi_1$

The angle between the vector $\mathbf{T}$ and the ort e_{ξ_1} will be denoted by Π. This is either the incidence angle $(\Pi > \pi/2)$ or reflection angle $(\Pi < \pi/2)$ of the ray in the wake. In the case when $\Pi = \pi/2$, a rotation of the ray takes place. In order to find the coordinates of the point where the rotation of the ray takes place, we shall make use of the formula

$$\cos(\vec{T}, \vec{e}_{\xi_1}) = \cos\Pi = \frac{\vec{T}\vec{e}_{\xi_1}}{|\vec{T}||\vec{e}_{\xi_1}|} = \sqrt{\frac{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}{\xi_1^4 - f\xi_1^2 + \xi_1^2\xi_2^2}}. \tag{10}$$

At the rotation point $\cos\Pi = 0$. Consequently, from the equation

$$\xi_{11}^4 - a_1^2\xi_{11}^2 - a_3^2 - f(\xi_{11})\xi_{11}^2 = 0 \tag{11}$$

we may find the coordinate ξ_{11} of the rotation point $P_1(\xi_{11}, \xi_{21}, \xi_{31})$.

As an example we shall find the value of ξ_{11} when

$$f = \begin{cases} b^2(\xi_{10}^2 - \xi_1^2), & \xi_1 \leqslant \xi_{10}, \\ 0, & \xi_1 > \xi_{10}. \end{cases}$$

Resolving (11), we shall obtain

$$\xi_{11}^2 = \frac{a_1 + b^2\xi_{10}^2}{2(1+b^2)} \pm \sqrt{\left[\frac{a_1^2 + b^2\xi_{10}^2}{2(1+b^2)}\right]^2 + \frac{a_3^2}{1+b^2}}.$$

If $a_3 = 0$, we have

$$\xi_{11}^2 = (a_1^2 + \xi_{10}^2 b^2) / (1+b^2), \quad \xi_{11} = 0.$$

The coordinates ξ_{21} and ξ_{31} are respectively determined from the first and second equations of the system (6) :

$$\xi_{21}^2 = \left[(Б^{(1)} - a_1^2)^2 + 4a_3^2 \right] / 4Б^{(1)}, \tag{12}$$

where

$$Б^{(1)} = K^{(1)} \exp\left[2M^{(1)}\right];$$

$$K^{(1)} = 2\sqrt{\xi_{20}^4 + a_1^2\xi_{20}^2 - a_3^2} + 2\xi_{20}^2 + a_1^2;$$

$$M^{(1)} = \int\limits_{\xi_{10}}^{\xi_{11}} \frac{\xi_1\, d\xi_1}{\sqrt{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}};$$

$$\arcsin \xi_{31} = \arcsin \xi_{30} + M^{(2)} + \\ + \frac{1}{2}\left\{ \arcsin \frac{2a_3^2 - a_1^2\xi_{20}^2}{\xi_{20}^2\sqrt{a_1^4 + 4a_3^2}} - \arcsin \frac{2a_3^2 - a_1^2\xi_{21}^2}{\xi_{21}^2\sqrt{a_1^4 + 4a_3^2}} \right\}, \tag{13}$$

where

$$M^{(2)} = \int\limits_{\xi_{20}}^{\xi_{11}} \frac{a_3\, d\xi_1}{\xi_1\sqrt{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}}.$$

After the point of rotation, the ray, emerging from the wake, will intersect its surface $\xi_1 = \xi_{10}$ at the point P_2 , of which the coordinates are

$$\xi_{12} = \xi_{10},$$

$$\xi_{22}^2 = \left[(Б^{(2)} - a_1^2)^2 + 4a_3^2 \right] / 4Б^{(2)}, \tag{14}$$

$$\arcsin \xi_{32} = \arcsin \xi_{30} + 2M^{(2)} + \tag{15}$$

$$+ \frac{1}{2}\left\{ \arcsin \frac{2a_3^2 - a_1^2\xi_{20}^2}{\xi_{20}^2\sqrt{a_1^4 + 4a_3^2}} - \arcsin \frac{2a_3^2 - a_1^2\xi_{22}^2}{\xi_{22}^2\sqrt{a_1^4 + 4a_3^2}} \right\}, \tag{16}$$

where

$$Б^{(2)} = K^{(1)} \exp\left[4M^{(1)}\right].$$

The ray propagates further in the free space in the direction of the vector $\vec{T}$.

2. - ABSORPTION OF THE ELECTROMAGNETIC ENERGY IN THE WAKE.

The density decrease of the electromagnetic energy flux at the expense of energy absorption in the ionized wake may be computed by the formula

$$S = S_0 \exp\left[-2k \int\limits_{l_0}^{l} \varkappa\, dl \right], \tag{17}$$

where S_o is the initial density of the flux; k is the wave number; $\varkappa$ is the absorption coefficient.

The integral under the exponent is taken along the ray. Usually, the absorption coefficient of the ionized medium at points where the ray passes, may be approximately represented in the form

$$\varkappa = \varkappa_0 f(\xi_1) / h^2(\xi_1, \xi_2) n, \quad \varkappa_0 = \nu_{\text{эф}} / 2\omega, \tag{18}$$

when $|\varepsilon| \gg 4\pi\sigma/\omega$, $\varepsilon > 0$. Here σ is the conductivity of the ionized medium; $\nu_{\text{эф}}$ is the effective collision frequency of electrons with ions; ω is the cyclical frequency of the incident wave; ε is the dielectric constant.

After the remarks made it is necessary to find the integral under the exponent in (17) that determines the value of the weakening.

The length element of the ray in the parabolic system of coordinates is

$$dl = \sqrt{h_1^2 + h_2^2\left(\frac{d\xi_2}{d\xi_1}\right)^2 + h_3^2\left(\frac{d\xi_3}{d\xi_1}\right)^2}\, d\xi_1.$$

Consequently,

$$I(\xi_{10}, \xi_1, a_1, a_3) = \int_{l_0}^{l} \varkappa dl = \int_{\xi_{10}}^{\xi_1} \varkappa \sqrt{h_1^2 + h_2^2\left(\frac{d\xi_2}{d\xi_1}\right)^2 + h_3^2\left(\frac{d\xi_3}{d\xi_1}\right)^2}\, d\xi_1.$$

Having determined from (6) the derivatives $d\xi_2/d\xi_1$ and $d\xi_3/d\xi_1$, and also taking into account (18), we shall obtain

$$I(\xi_{10}, \xi_1, a_1, a_3) = \int_{\xi_{10}}^{\xi_1} \frac{\varkappa_0 f(\xi_1)\, \xi_1\, d\xi_1}{\sqrt{\xi_1^4 - a_1^2\xi_1^2 - f(\xi_1)\,\xi_1^2 - a_3^2}}. \tag{19}$$

At ray's passage through the wake, the density of the flux has its maximum weakening

$$I_{max} = 2\, I\, (\xi_{10}, \xi_{11}, a_1, a_3). \tag{20}$$

Therefore, (19) determines the value of flux density decrease at the expense of energy loss in the ionized medium in the direction of propagation of the ray.

3. - RADAR REFLECTION OF THE RAY FROM THE WAKE

We shall consider all the possible cases of radar detection of the ray. The wake may be detected by radar so long as the ray is incident and is reflected from the surface of the wake along the normal to the surface. This means that

$$\cos \Pi = 1.$$

Taking into account (10), we shall have

$$\sqrt{\frac{\xi_1^4 - a_1^2\xi_1^2 - a_3^2 - f\xi_1^2}{\xi_1^4 - f\xi_1^2 + \xi_1^2\xi_2^2}} = 1.$$

The last equality is fulfilled, at first when $f \to \infty$, which implies an infinite increase of electron concentration in the wake, the rays then reflecting from the surface of the wake as if it were metallic, and secondly, when $-\xi_1^2(a_1^2 + \xi_2^2) = a_3^2$, which is fulfilled only at $a_3 = 0$.

If $a_3 = 0$, the trajectory of the ray must lie in the plane $\xi_3 = 1$.

Moreover, radar reflection of the ray from the wake is possible when the condition

$$\cos \Pi_{P_0} = -\cos \Pi_{P_2}.$$

is satisfied at wake boundary. Consequently, according to (10) we must have

$$\xi_{22}^2 = \xi_{20}^2 \quad \text{at} \quad a_3 = 0. \tag{21}$$

Substituting (21) into (15), where $a_3 = 0$, we shall obtain

$$\xi_{20} = (B^{(2)} - a_1^2) \,/\, 2B^{(2)1/2}. \tag{22}$$

Let us consider an example. Assume that

$$f(\xi_1) = b^2(\xi_{10}^2 - \xi_1^2);$$

Then

$$M^{(1)} = \frac{1}{2\sqrt{1+b^2}} \ln \frac{\xi_{10}^2 b^2 + a_1^2}{(\xi_{10}\sqrt{1+b^2} + \sqrt{\xi_{10}^2 - a_1^2})^2}. \tag{23}$$

Consequently

$$B^{(2)} = K^{(1)}\left[\frac{\sqrt{\xi_{10}^2 b^2 + a_1^2}}{\xi_{10}\sqrt{1+b^2} + \sqrt{\xi_{10}^2 - a_1^2}}\right]^{4/\sqrt{1+b^2}} = K^{(1)}A_1.$$

Taking into account the results obtained in the example brought out, and effecting subsequent transformations of (22), we shall have

$$\cos H_0 = \frac{\xi_{20}}{h_{20}} \frac{1 - A_1^{1/2}}{1 + A_1^{1/2}}. \tag{24}$$

Formula (24) determines the direction of the ray at the chosen point $P_2(\xi_{10}, \xi_{20}, 1)$, at which radar detection of the ray is possible.

In a particular case it follows from (24) that at $b^2 \to \infty$ $A_1 \to 1$, and $\cos H_0 \to 0$. Consequently, at infinite increase in the density of free electrons in the wake $H_0 \to \pi/2$, that is, the rays, incident on wake surface along the normal to the surface, are reflected strictly backward.

If $b^2 < \infty$, the direction of H_0, from which the wake is detected by radar, constitutes the solution of the transcendential equation (24).

Formula (24) may also be written for the arbitrary function $f(\xi_1)$. Then the quantity A_1 in (24) must be substituted by

$$A_1 = \exp\,[4M^{(1)}].$$

4. - EFFECTIVE REFLECTING SURFACE

The ray crosses the surface of the wake at the point P_3, of which the Descartes coordinates are

$$x_2 = \xi_{12}\xi_{22}\xi_{32}, \quad y_2 = \xi_{12}\xi_{22}\sqrt{1 - \xi_{32}^2}, \tag{25}$$
$$z_2 = \frac{1}{2}(\xi_1^2 - \xi_2^2).$$

The equation of the ray passing through the point P_2 and propagating in the direction of the vector $\vec{T}$ will be written in the form

$$(x - x_2)\,/\,T_x = (y - y_2)\,/\,T_y = (z - z_2)\,/\,T_z. \tag{26}$$

This ray crosses the sphere

$$x^2 + y^2 + z^2 = R^2 \tag{27}$$

at the point P_3, whose coordinates are determined by the system of equations (26) and (27), and are equal to

$$x_3 = x_2 - A + B, \qquad y_3 = \frac{T_y}{T_x}(x_3 - x_2) + y_2, \quad z_3 = \frac{T_z}{T_x}(x_3 - x_2) + z_2, \tag{28}$$

where

$$A = T_x(\vec{T_0}, \vec{r_2});$$

$$B = \sqrt{(x_2 - A)^2 + T_x^2 R^2 - (-T_y x_2 + T_x y_2)^2 - (-T_z x_2 + T_x z_2)^2};$$

T_x, T_y, T_z are the projections of T on the axis of the coordinates. The coordinates of the point P_3 depend on the point P as of the parameter. That is why the coordinates z, y of the point P are curvilinear coordinates on a sphere of radius R.

Denoting

$$g_{zy} = \frac{\partial x_3}{\partial z}\frac{\partial x_3}{\partial y} + \frac{\partial y_3}{\partial z}\frac{\partial y_3}{\partial y} + \frac{\partial z_3}{\partial z}\frac{\partial z_3}{\partial y} \tag{29}$$

and determining analogously g_{zz} and g_{yy}, we may write the surface element on the sphere in the form [3]

$$ds_2 = \sqrt{g}\,dz\,dy, \tag{30}$$

where

$$g = g_{zz} g_{yy} - g_{zy}^2.$$

Therefore, the beam of rays, resting on the front of the wave incident upon the area $ds_1 = dl\,dy = dz\,dy / \cos\alpha$, will cut out on the surface of the sphere an area ds_2, the direction of the normal to which coincides with the direction of the radius-vector. Consequently,

$$S = S_0 \frac{1}{\cos\alpha(\vec{T_0}\vec{r_2})\sqrt{g}}. \tag{31}$$

When $r_2 \ll R$, formula (31) is simplified. We may write approximately

$$S = S_0 \frac{1}{\cos\alpha R^2 \sqrt{g_0}} \tag{32}$$

where

$$g_0 = g_{0zz} g_{0yy} - g_{0zy}^2;$$

$$g_{0zy} = \frac{\partial x_{03}}{\partial z}\frac{\partial x_{03}}{\partial y} + \frac{\partial y_{03}}{\partial z}\frac{\partial y_{03}}{\partial y} + \frac{\partial z_{03}}{\partial z}\frac{\partial z_{03}}{\partial y};$$

g_{0zz} and g_{0yy} are determined analogously. Here the quantities x_{03}, y_{03}, z_{03} are projections on the axis of coordinates of the ort of the vector from the point P_2 to the point P_3. However, when $r_2 \ll R$, they are equal to directional cosines. According to (28)

$$x_{03} = \frac{x_3}{R} = T_{0x} + o\left(\frac{r_2}{R}\right),$$
$$y_{03} = \frac{y_3}{R} = T_{0y} + o\left(\frac{r_2}{R}\right)$$
$$z_{03} = \frac{z^3}{R} = T_{0z} + o\left(\frac{r_2}{R}\right).$$

The quantities T_{0x}, T_{0y}, T_{0z} are projections of $\vec{T}_0$ in the rectilinear system of coordinates, which may be expressed through the respective projections of $\vec{T}_0$ in the parabolic system by the formula

$$A_j = \sum_{i=1}^{3} a_i h_i^{-1} \frac{\partial x_j}{d\xi_i}.$$

At the point P_2 they are

$$T_{0x} = \frac{V_{10}\xi_{22}\xi_{32}}{\xi_{10}h_{22}^2} + \frac{V_{20}\xi_{12}\xi_{32}}{\xi_{22}h_{22}^2} + \frac{a_3}{h_{32}},$$

$$T_{0y} = \frac{V_{10}\xi_{22}\sqrt{1-\xi_{32}^2}}{\xi_{10}h_{22}^2} + \frac{V_{20}\xi_{12}\sqrt{1-\xi_{32}^2}}{\xi_{22}h_{22}^2} - \frac{a_3\xi_{32}}{\xi_{12}\xi_{22}}$$

$$T_{0z} = \frac{V_{10}}{h_{22}^2} - \frac{V_{20}}{h_{22}^2},$$

where

$$V_{10} = \sqrt{\xi_{10}^4 - \xi_{10}^2 a_1^2 - a_3^2}; \quad V_{20} = \sqrt{\xi_{20}^4 + \xi_{20}^2 a_1^2 - a_3^2};$$
$$h_{22} = \sqrt{\xi_{12}^2 + \xi_{22}^2}.$$

It stems from formula (32) that the effective reflecting surface of the wake is

$$\sigma = 4\pi / \cos\alpha(\vec{T}_0\vec{n})\sqrt{g_0}. \qquad (33)$$

The quantity σ depends on the propagation direction of the ray having passed through the plasma wake, that is from the vector $\vec{T}_0$ which in its turn depends on the propagation direction of the incident ray and on the coordinates of the point P lying in the plane of the incident ray front. If the propagation direction of the incident ray is given, $\vec{T}_0$ will be determined by the position of P and consequently, it is dependent on z, y as of Gaussian coordinate parameters. The cosine of the angle between the coordinate lines $z =$ const and $y =$ const on a sphere of radius R is $\cos(\widehat{z, y}) = g_{0zy}$. Hence it may be seen that when $g_{0zy} = 0$, the coordinate lines on the sphere are orthogonal. This condition is filfilled at $a_3 = 0$.

5. - SCATTERING OF RAYS IN THE PLANE $\xi_3 = 1$

As an example of application of the formulas derived we shall consider the reflection of a plane wave from an ionized wake in the plane $\xi_3 = 1$

In this case, according to (7), $a_3 = 0$. In order to determine σ, it is necessary to find the derivatives of T_{0xz}, T_{0yz}, T_{0zz} and T_{0xy}, T_{0yy}, T_{0zy}. At $\xi_3 = 1$ and $a_3 = 0$, these derivaties are respectively

$$T_{0xz} = \cos\alpha \left[\frac{\xi_{22}}{h_{22}^2} - \frac{\xi_{12}\xi_{22}}{h_{22}V_{22}}\Pi\right]\xi_{20z} +$$

$$+\left[\Pi\left(1 - \frac{\xi_{22}^2}{h_{22}^2}\right)\frac{1}{h_{22}^2} + \frac{\xi_{12}\xi_{22}^2}{h_{22}^2V_{22}} - \frac{V_{22}}{h_{22}^3}\xi_{12}\xi_{22}\right]\xi_{22z},$$

$$T_{0yz} = 0,$$

$$T_{0zz} = \cos\alpha\left[\frac{\xi_{10}}{h_{22}^2} + \frac{\xi_{22}\Pi}{h_{22}^2V_{22}}\right]\xi_{20z} -$$

$$-\left[\frac{\xi_{10}\xi_{22}\Pi}{h_{22}^3} + \frac{\xi_{22}^2}{h_{22}^2V_{22}} + \frac{V_{22}}{h_{22}^2}\left(1 - \frac{\xi_{22}^2}{h_{22}}\right)\right]\xi_{22z},$$

$$T_{0xy} = 0,$$

$$T_{0yy} = \left(\frac{V_{12}\xi_{22}}{\xi_{10}h_{22}^2} + \frac{V_{22}\xi_{10}}{h_{22}^2\xi_{22}}\right)\frac{1}{\xi_{10}\xi_{20}},$$

$$T_{0zy} = 0.$$

Here

$$\Pi = \xi_{10}\sin\alpha + \xi_{20}\cos\alpha;$$

$$\xi_{20z} = \partial\xi_{20} / \partial z = -1 / \xi_{20};$$

$$\xi_{22z} = \frac{\xi_{20z}}{2}A_1^{1/2}\left\{1 - \sin\alpha + \frac{BA_{1\xi20}}{2A_1} - \frac{a_1^2}{B^2A_1}(1 - \sin\alpha) + \frac{a_1^2A_{1\xi20}}{2BA_1^2} +\right.$$

$$\left. + \frac{2\xi_{20}\cos^2\alpha + \xi_{10}\sin 2\alpha}{BA_1}\right\};$$

$$B = \xi_{20}(1 - \sin\alpha) + \xi_{10}\cos\alpha; \qquad A_{1\xi20} = dA_1 / d\xi_{20}.$$

It may be seen from (34) that $g_{0zy} = 0$ at $\xi_3 = 1$ and $a_3 = 0$. Thus,

$$\sigma = \frac{4\pi}{\cos\alpha(\vec{T}_0\vec{n})T_{0yy}\sqrt{T^2_{0xz} + T^2_{0zz}}}.$$

If the irradiation of the wake by the radar station takes place from the forward hemisphere ($\xi_{20} = 0$, $\cos\alpha = 0$), the effective reflecting surface is

$$\sigma = \pi\xi_{10}^4A_1^{1/2}.$$

It follows also from this formula that at $b^2 \to \infty$,

$$\sigma = \pi\xi_{10}^4,$$

which coincides with the reflecting surface of a metallic paraboloid [4].

If b → 0, σ → 0. If the irradiation of the wake by the radar station takes place laterally

$$\sigma=\frac{8\pi\xi_{10}^2\xi_{20}^2A_1^{1/2}}{(1-A_1^{1/2})^2+\dfrac{2\xi_{10}[(1+A_1)\xi_{10}+(1-A_1)\tilde{\xi}_{20}]}{\xi_{10}^2b^2+a_1^2}}$$

where $\tilde{\xi}_{20}$ is determined from the formula

$$\frac{\tilde{\xi}_{20}}{\xi_{10}}=\frac{1+A_1^{1/2}}{1-A_1^{1/2}}.$$

Therefore, in order to be able to investigate the propagation of radiowaves in an ionized wake it is appropriate to utilize a parabolic system of coordinates of rotation, which allows to find the trajectories of of the rays inside the wake, the energy absorption of wake's plasma and to determine the effecting reflecting surface of the wake for a rather arbitrary distribution function of free element concentration.

**** THE END ****

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